STABILITY OF A SYSTEM OF PARALLEL BOILING CHANNELS

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Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 3, pp. 300-306, 1965

The stability of the equilibrium steady-state regime is examined "in the small" for a system of parallel boiling channels connected to common headers. The necessary and sufficient conditions of stability are obtained.

A number of papers [1-3] have been devoted to the conditions of existence of instability in a system of parallel steam-generating channels. The present paper gives a solution of the problem of the stability of identical boiling channels, connected in parallel, the headers joining them, and the external parts of the network, – supply pipes, – (the "source") and a load (steampipes, turbines, condensers, etc.) (Fig. 1).

Neglecting the inertial properties of the external parts of the network, we may represent them as equivalent lumped resistances (inlet and outlet).

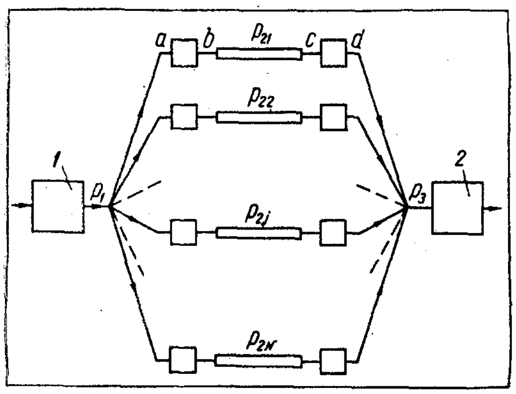

Fig. 1. Block diagram of the system of parallel boiling channels and the external parts of the network: 1), 2) equivalent lumped resistances (inlet and outlet).

The pressure drop in the heated part of the channels (bc) will be considered negligibly small compared with that in the unheated parts (ab) and (cd). We shall represent parts (ab) and (cd) by equivalent lumped resistances.

The heated part of the channel is assumed to be long enough for the liquid heat-transfer agent, which arrives at the channel entrance at constant temperature, to be heated and evaporated and leave the channel in vapor form.

The system of dynamic equations of section (bc) will then consist of three groups, describing the dynamics of the economizer, evaporator, and superheater zones, and a system of boundary conditions determining the continuity of variation of the parameters of the heat-transfer agent as it passes from zone to zone.

We shall describe the dynamics of each zone by a system of heat balance and continuity equations

$$\frac{\partial}{\partial x} Gi + S \frac{\partial}{\partial t} \gamma i = Sq,$$
$$\frac{\partial}{\partial x} G + S \frac{\partial}{\partial t} \gamma = 0. \quad (1)$$

We shall also assume that the temperature of the external heater, the heat transfer coefficient, density and heat capacity of the liquid and vapor, and flow cross section of the heated part of the channels are constant, and that the heat capacity of the channel walls is negligibly small.

Let us examine one of the channels of the system. With the above assumptions, the dynamic equations of the heated section of the channel may be written as follows:

Economizer Zone

$$\frac{\partial}{\partial x} w_l i_l + \frac{\partial}{\partial t} i_l = \frac{k}{\gamma'} [\vartheta_1 - \vartheta_l (x, t)],$$
$$0 \leqslant x \leqslant h_{ec}(t), w_l (t) = w_{in}(t), \ i_l = c \vartheta_l, \quad (2)$$
$$i_l (0, t) = i_{in} = \text{const}, \ i_l [h_{ec}(t), t] = i_s (t).$$

Linearizing (2) [4] and using the Laplace transform method with zero initial conditions [5], we obtain

$$\overline{\Delta h}_{ec} = \left(\frac{di_s}{dp_2}\right)_0 \frac{w_{in.o}\ \gamma'}{k(\vartheta_1 - \vartheta_s)_0}\ \overline{\Delta p_2} + \frac{1}{\lambda}[1 - \exp(-\lambda\tau_{ec})]\overline{\Delta w_{in}}, \tag{3}$$

where $\tau_{ec} = \frac{c\gamma'}{k} \ln \frac{\vartheta_1 - \vartheta_{in}}{\vartheta_1 - \vartheta_{s0}}$ is the time taken by the heat transfer agent to pass through the economizer zone.

Evaporator Zone

$$\frac{\partial}{\partial x}[w_v\gamma''\varphi i_v + w_l\gamma'(1-\varphi)i_s] + \frac{\partial}{\partial t}[\gamma''\varphi i_v + \gamma'(1-\varphi)i_s] = q,$$

$$\frac{\partial}{\partial x}[w_v\gamma''\varphi + w_l\gamma'(1-\varphi)] + \frac{\partial}{\partial t}[\gamma''\varphi + \gamma'(1-\varphi)] = 0, \tag{4}$$

$$w_l[h_{ev}(t), t] = w_{in},\ w_v = w_v(w_l, p_2, \varphi, \ldots),\ q = k[\vartheta_1 - \vartheta_s(t)],$$

$$\varphi[h_{ec}(t), t] = 0,\ \varphi[h_{ev}(t), t] = 1,\ h_{ec}(t) \leqslant x \leqslant h_{ev}(t).$$

The state of the flow of heat transfer agent in the evaporator zone will be characterized by the mean volume steam content for the zone φ_{mo}, determined from (4) for the conditions of the initial equilibrium steady-state regime (φ_{mo} = const). Integrating (4) with respect to x over the interval $h_{ec}(t) \le x \le h_{ev}(t)$, linearizing the resulting equation, going over to Laplace transform variables with zero initial conditions, and taking (3) into account, we finally obtain

$$\frac{\overline{\Delta w_v(h_{ev})}}{w_{v0}} = -L_1(\lambda)\overline{\Delta p_2} + L_2(\lambda)\frac{\overline{\Delta w_{in}}}{w_{in.o}}, \tag{5}$$

where

$$L_1(\lambda) = \frac{B(1-\sigma)\lambda\tau_1}{1+\lambda\tau_1}[\lambda\tau_1(1+A) + \psi],$$

$$L_2(\lambda) = \sigma + \frac{(1-\sigma)\exp(-\lambda\tau_{ec})}{1+\lambda\tau_1},$$

$$A = \sigma\frac{\varphi_{mo}}{1-\varphi_{mo}}\left(\frac{di_v}{di_s}\right)_0,\quad B = \left(\frac{di_s}{dp_2}\right)_0 \frac{1}{r},$$

$$\tau_1 = \frac{(1-\varphi_{mo})\gamma' r}{k(\vartheta_1 - \vartheta_s)_0},$$

$$\psi = \frac{1}{1-\varphi_{mo}}\left[\varphi_{mo} + (1-\varphi_{mo})\left(\frac{di_v}{di_s}\right)_0\right],\quad \sigma = \frac{\gamma''}{\gamma'}.$$

Replacing $\overline{\Delta w_v(h_{ev})}$ by $\overline{\Delta w_v(H)} = \overline{\Delta w_{ex}}$ in (5) (the validity of this substitution follows from the assumed constant density of the vapor in the part of the superheater zone $h_{ev}(t) \leqslant x \leqslant H$) we obtain the dynamic equation of the heated section of the boiling channel:

$$\frac{\overline{\Delta w_{ex}(H)}}{w_{ex.o}} = -L_1(\lambda)\overline{\Delta p_2} + L_2(\lambda)\frac{\overline{\Delta w_{in}}}{w_{in.o}}. \tag{6}$$

The linearized hydraulic equations of the unheated sections of the channels, the source, and the load have the form

$$\overline{\Delta p_1} - \overline{\Delta p_3} = \frac{2}{N}\left[\sum_{j=1}^{N} U_{1j}\frac{\overline{\Delta w_{in.j}}}{(w_{in.j})_0} + \sum_{j=1}^{N} U_{2j}\frac{\overline{\Delta w_{ex.j}}}{(w_{ex.j})_0}\right],$$

$$\overline{\Delta p_{2j}} = \overline{\Delta p_1} - 2U_{1j}\frac{\overline{\Delta w_{in.j}}}{(w_{in.j})_0}, \tag{7}$$

$$-\overline{\Delta p_1} = \frac{2V_1}{N^3}\sum_{j=1}^{N}\frac{\overline{\Delta w_{in.j}}}{(w_{in.j})_0}, \tag{8}$$

$$\overline{\Delta p_3} = \frac{2V_2}{N^3} \sum_{j=1}^{N} \frac{\overline{\Delta w}_{ex \cdot j}}{(w_{ex.j})_0}, \tag{9}$$

where

$$V_1 = -\frac{1}{2} G_0^* \left(\frac{\partial p_1}{\partial G^*}\right)_0, \quad V_2 = \frac{1}{2} G_0^* \left(\frac{\partial p_3}{\partial G^*}\right)_0,$$

$$U_{1j} = \frac{1}{2} G_{j0} \left[\frac{\partial (p_1 - p_{2j})}{\partial G_j}\right]_0, \; U_{2j} = \frac{1}{2} G_{j0} \left[\frac{\partial (p_{2j} - p_3)}{\partial G_j}\right]_0,$$

$$G_0^* = \sum_{j=1}^{N} G_{j0}, \quad j = 1, 2, 3, \dots, N.$$

If the heat transfer agent reaches the inlet header through a pipe connected to the main in which the pressure p_0 = = const and if this pipe, the unheated sections of the channels, and the load are turbulent resistances (turbulent self-similar flow regime), then $V_1 = (p_0 - p_1)_0$, $V_2 = (p_3 - p_4)_0$, $U_{1j} = (p_1 - p_{2j})_0$, $U_{2j} = (p_{2j} - p_3)_0$, where $(p_3 - p_4)_0$ is the steady-state pressure drop at the load.

Characteristic Equation. Stability.

The hydraulic characteristics of the unheated sections (ab) and (cd) and the ϑ_{1j} will be assumed to be identical for all N channels. Then the mass flow of the heat transfer agent, the length of the zones, and the values of all the other parameters characterizing the equilibrium steady-state regime will not depend on the channel number j. In this case the characteristic equation of system (6-9) will have the form

$$(F_1)^{N-1} F_2 = 0,$$

$$F_m = 2L_1 \Phi_{1m} + L_2 \Phi_{2m} + \Phi_{1m},$$

where

$$m = 1,2; \; \Phi_{11} = U_1, \; \Phi_{21} = U_2,$$

$$\Phi_{12} = \frac{V_1}{N^2} + U_1, \quad \Phi_{22} = \frac{V_2}{N^2} + U_2,$$

F_1 determines the stability of the system ("in the small") in relation to so-called interchannel (inter-loop) pulsation, and F_2 in relation to general boiler pulsation*. It is thus necessary and sufficient for the stability of the system under examination to achieve stability with respect to both types of possible pulsations: interchannel and general boiler.

Since the operators F_1 and F_2 differ only in the values of the parameters, to determine the conditions of steady-correction made has created more space below insert 303 e directly above. Please redistribute this space.

$$2L_1 \Phi_1 \Phi_2 + L_2 \Phi_2 + \Phi_1 = 0. \tag{10}$$

Carrying out a D-partition [6-7] of the plane of the parameters Φ_2 and Φ_1, we find that all the roots of (10) have a negative real part if Φ_2 and Φ_1 lie in region D_0, located in the first quadrant of the (Φ_2, Φ_1) plane, and above the curve determined by the following equations:

$$\begin{aligned} \Phi_2 = {} & (\sin \omega\tau_{ec} + \omega\tau_1 \cos \omega\tau_{ec}) \times \\ & \times [2B\omega\tau_1 \{(1-\sigma)\omega\tau_1(1+A)\sin \omega\tau_{ec} - \sigma[\psi + (\omega\tau_1)^2(1+A)] - \\ & - (1-\sigma)\psi \cos \omega\tau_{ec}\}]^{-1}, \\ \Phi_1 = {} & \frac{\sin \omega\tau_{ec} + \omega\tau_1 \cos \omega\tau_{ec}}{2B\omega\tau_1[\psi + (\omega\tau_1)^2(1+A)]}, \quad \frac{\pi}{2} \leqslant \omega\tau_{ec} \leqslant \pi. \end{aligned} \tag{11}$$

*In the case of general boiler pulsation, fluctuations in the mass flow of heat transfer agent occur synchronously in the individual channels. "Interchannel" pulsation is characterized by constant mass flow of heat transfer agent through the source and load and asynchronous fluctuations in the individual channels.

It is therefore necessary and sufficient for the stability of the system that points (Φ_{21}, Φ_{11}) and (Φ_{22}, Φ_{12}) belong to the region D_0, i. e.,

$$(\Phi_{21}, \Phi_{11}), (\Phi_{22}, \Phi_{12}) \in D_0. \qquad (12)$$

When N = 1, $(F_1)^{N-1} \equiv 1$, so the stability condition for a single channel has the form

$$(\Phi_{22}, \Phi_{12}) \in D_0.$$

It follows from (12) that

1) the stability of the equilibrium steady-state regime depends appreciably on the hydraulic characteristics of the channels, their number, and the characteristics of the external parts of the network;

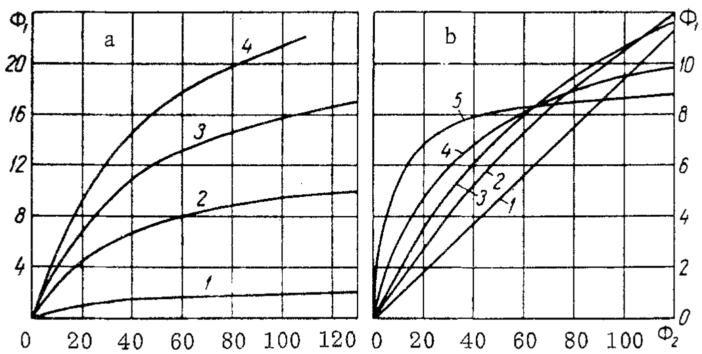

Fig. 2. Boundary of stability region D_0:
a) for φ_{mo} = 0.85 (1, 2, 3, 4 – ϑ_i = 512, 472, 432, 392°K, respectively);
b) for ϑ_{in} = 472°K (1, 2, 3, 4, 5 – φ_{mo} = 0.09, 0.47, 0.65, 0.85, 1 respectively).

2) the stability of the system with respect to interchannel pulsations is not, in general, determined by its stability with respect to general boiler pulsations (the special case of $V_2 = 0$ examined in [3] is an exception);

3) there is a critical value of the parameter $U_1^* = U_1^* (p_{20}, i_{in}, k, \vartheta_1)$, such that for all $U_1 > U_1^*$, the system will be stable for any N, V_1, V_2 and U_2;

4) the period of the oscillations developing in the system on passing through the boundary of the stability region (11) is of the same order as the time taken by the heat transfer agent to pass through the economizer zone.

The location of the boundary of the stability region D_0 for the case in which the heat transfer agent is water at p_{20} = 3.92 Mn/m² and q = const, is shown in Fig. 2.

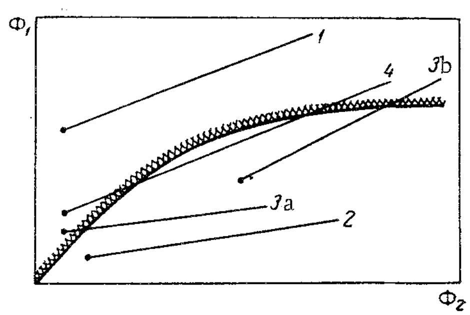

Fig. 3. Variants of the relative location of line segment (13) and curve (11) – 1, 2, 3a, 3b, 4.

The results of similar calculations show that for other values of the pressure p_{20} ($0.28 \leqslant p_{20} \leqslant 9.8$ Mn/m²) the shape of the region D_0 and the dependence of the location of its boundary on φ_{mo} and ϑ_i are qualitatively similar to those in Fig. 2.

The dependence of the stability of the system on the number of channels is of interest. Let us take an example. Assume that V_1 and V_2 do not depend on N. For $V_1 = (p_0 - p_1)_0$ and $V_2 = (p_3 - p_4)_0$ the condition V_1 = const, V_2 = const indicates that, as the number of channels changes, the hydraulic characteristics of the supply line and load also change, in such a way that the steady-state pressure drops $(p_0 - p_1)_0$, $(p_3 - p_4)_0$ and the quantities p_0 and p_4 remain unchanged. Note that for V_1 = const and V_2 = const

$$\lim_{N\to\infty} \Phi_{22}(N) = \Phi_{22}(\infty) = \Phi_{21}, \quad \lim_{N\to\infty} \Phi_{12}(N) = \Phi_{12}(\infty) = \Phi_{11}.$$

Hence the conditions of stability of the steady-state regime may be written in the form *

$$[\Phi_{22}(N), \Phi_{12}(N)] \in D_0,$$
$$[\Phi_{22}(\infty), \Phi_{12}(\infty)] \in D_0.$$

*As was pointed out, for N = 1 the stability condition may be written $[\Phi_{22}(1), \Phi_{12}(1)] \in D_0$.

When N varies from 1 to infinity, the points $[\Phi_{22}(N), \Phi_{12}(N)]$ lie on the line segment:

$$\Phi_1 = \frac{V_1}{V_2}\Phi_2 + \Phi_{12}(\infty) - \frac{V_1}{V_2}\Phi_{22}(\infty). \qquad (13)$$

$$\Phi_{22}(\infty) \leqslant \Phi_2 \leqslant V_2 + \Phi_{22}(\infty).$$

Depending on the relative location of segment (13) and curve (11) in the plane (ϕ_2, ϕ_1), the following possibilities, in particular, may be realized (see Fig. 3):

1) If segment (13) lies entirely above curve (11), the system is stable for any N.

2) If segment (13) lies entirely below curve (11), the system is unstable for any N.

3) Segment (13) intersects curve (11) once. Then:

a) if the right end of the intercept lies below the curve, a number m_1 exists such that, for all $N > m_1$, the system is stable, but it is unstable for $N < m_1$ (for $N < m_1$ the stability condition with respect to general boiler pulsations is disturbed, but the system remains stable in the small with respect to interchannel pulsations);

b) if the left end of the intercept lies below the curve, then for N = 1 the system is stable, but it is unstable for $N \geq 2$ (for $N \geq 2$ the stability condition with respect to interchannel pulsations is disturbed; the system may be stable with respect to general boiler pulsations).

4) If segment (13) cuts curve (11) twice, the system is either stable for any N, or numbers m_2 and m_3 exist, such that for $m_2 < N < m_3$ the system is unstable, and for $N < m_2$ and $N > m_3$ it is stable (for $m_2 < N < m_3$ the stability condition with respect to general boiler pulsations is disturbed).

NOTATION

p_1, p_2, and p_3 – pressure at header inlet, in heated part of channel, and at header outlet; G – mass flow rate of heat transfer agent; $p_1(G^*)$ and $p_3(G^*)$ – functions determining the dependence of pressure in the headers on the mass flow rate of heat transfer agent in sections 1 and 2, respectively; j – number of boiling channel; w – velocity of heat transfer agent; γ – density of heat transfer agent; γ' – density of liquid; γ'' – density of vapor; q – heat flux to unit volume of heat transfer agent; c – heat capacity of liquid; ϑ_1 – temperature of external heater; ϑ_i and ϑ_s – temperature of heat transfer agent at channel inlet and at saturation line; i – enthalpy; Δ – deviation of a variable from its steady-state value; φ – fraction of flow cross section occupied by vapor; φ_{mo} – mean volume steam content over evaporation zone; S – cross section of channel; x – coordinate calculated from beginning of heated section; h_{ec} – coordinate of boundary between economizer and evaporator zones; h_{ev} – coordinate of boundary between evaporator and superheater zones; H – length of heated section of channel; t – time; λ – Laplace transform parameter. Subscripts: 0 – refers to values of variables in the equilibrium steady-state regime in the neighborhood of which linearization is carried out; l – liquid heat transfer agent; v – vapor; in – inlet; ex – outlet.

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12 May 1964

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